

Seeking Lorentz Violation from the Higgs

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Abstract

The recently discovered Higgs particle with a mass near 126 GeV presents new opportunities to explore Lorentz violation. Ultra-high-energy cosmic rays are one of the most sensitive testing grounds for Lorentz symmetry, and can be used to seek for and limit departures from Lorentz invariance in the Higgs sector. If the Higgs were to have a super- or sub-luminal maximal speed both Higgs and weak interaction physics would be modified. Consideration of such modifications allow us to constrain the Higgs maximal velocity to agree with that of other Standard Model particles to parts in 10^{14} .

INTRODUCTION AND CONCLUSIONS

Departures from Lorentz invariance have been searched for in a wide variety of circumstances and limited with sometimes astonishing precision. Some of the best constraints arise from the novel kinematics associated with departures from a Lorentz invariant dispersion relation. In a Lorentz invariant world all particles share a universal maximal speed, usually referred to as the speed of light and conventionally chosen to be one. The energy and momentum of particle species i satisfy a Lorentz invariant dispersion relation $E^2 = p^2 + m_i^2$, and energy and momentum conservation then forbid many processes; for example, an electron (in vacuum) may not spontaneously radiate a photon, nor can a photon decay into an electron-positron pair.

Absent Lorentz invariance, as within a medium, the maximal speeds of different particle species may differ, $E^2 = c_i^2 p^2 + m_i^2$. Under these circumstances processes that are otherwise forbidden may be allowed. For example, in water the maximal speed of photons is slightly less than it is in vacuum and also less than that of electrons in the same medium ($c_\gamma < c_e$). In this case the emission of photons from superluminal (in the medium) electrons is both allowed and observed, where it is called Cerenkov radiation. If the vacuum itself is not Lorentz invariant a similar emission phenomenon can occur even in the absence of a medium. The observation of ultra-high-energy cosmic rays from distant sources limits the energy loss that would accompany this “vacuum Cerenkov radiation”, and provides many of the astonishing constraints referred to above[1, 2].

The recently discovered Higgs particle provides a new testing ground for Lorentz violation. Data from the LHC and elsewhere strongly favors the interpretation of this 126 GeV mass particle as the radial mode of a complex $SU(2)$ doublet field, H , where the remaining three fields of the doublet are “eaten” via the Higgs mechanism to give mass to the W and Z bosons. Adopting this interpretation we consider modifications of the Standard Model Lagrangian involving this doublet. Imposing rotational, CPT and CP invariance, we find that there is a unique (subject to field redefinitions) modification of the Lagrangian with operators of dimension four:

$$\mathcal{L}_{LV} = \delta D_0 H^\dagger D^0 H = \delta n \cdot DH^\dagger n \cdot DH . \quad (1)$$

where n^μ is a unit timelike four vector and δ is a parameter characterizing the size of Lorentz violation. This addition to the SM Lagrangian affects the propagation of both the physical

Higgs particle and the W and Z bosons:

- The maximal speed of the physical Higgs particle is $1/\sqrt{1+\delta}$. For positive δ the Higgs is subluminal; for negative δ it is superluminal.
- While the propagation of the two transverse polarizations of the W and Z bosons are unaffected, the third polarization has the same maximal speed as the Higgs particle, $1/\sqrt{1+\delta}$. The full vector boson propagator (in unitary gauge) is

$$\frac{-i}{k^2 - M^2} \left\{ g^{\mu\nu} - \frac{k^\mu k^\nu}{M^2} + \frac{\delta M^2}{k^2 - M^2 + \delta(k \cdot n)^2} \left[n^\mu n^\nu + (k \cdot n)^2 \frac{k^\mu k^\nu}{M^4} - \frac{(k \cdot n)}{M^2} (k^\mu n^\nu + k^\nu n^\mu) \right] \right\} \quad (2)$$

Observations of ultra-high-energy cosmic rays significantly constrain the size of these modifications. For positive δ the Higgs particle's maximal speed is subluminal and stable particles such as the proton would lose energy via “vacuum Higgs radiation”. The observation of high energy protons from distant sources limits such energy loss. Higgs emission from a high energy proton is kinematically allowed when the proton energy is greater than the threshold energy $E_0 = M_H/\sqrt{\delta}$ (ignoring small corrections from the proton mass) and the rate of this emission is readily calculated for small δ . At leading order in δ the amplitude for a proton to emit a Higgs with energy and momentum $q = (\epsilon, \mathbf{q})$ remains Lorentz invariant, and is then a function of the Higgs four-momentum squared $q^2 = \epsilon^2 - |\mathbf{q}|^2$. Using the dispersion relation for the subluminal Higgs this is $q^2 = -\delta(\epsilon^2 - E_0^2) \equiv -Q^2$. For Higgs energies a few times the threshold energy E_0 , q^2 is spacelike and much greater than the QCD scale. Since Q^2 is large the proton is likely to fragment as a result of the emission and the parton model may then be used to calculate the rate for the inclusive emission process *proton* \rightarrow *Higgs* + *hadrons*.

The rate for inclusive Higgs emission from a proton with momentum P is given by the incoherent sum of the rate for Higgs emission from each parton species f carrying momentum fraction x of the initial proton weighted by the parton distribution function $f_f(x)$ (ignoring the small effects of the proton mass and QCD corrections)

$$\Gamma(P) = \sum_f \int_0^1 dx f_f(x) \hat{\Gamma}_f(xP) \quad (3)$$

The Higgs interacts directly with quarks through the Yukawa couplings, and indirectly with gluons through the top quark loop. The contribution to the emission rate from quark q is

$$\Gamma = \frac{M_H^2}{48\pi E} \left(\frac{E}{E_0} \right)^2 \lambda_q^2 \int_{\frac{E_0}{E}}^1 dx x f_q(x) \left(1 - \frac{E_0}{E} \frac{1}{x} \right)^2 \left(1 + \frac{E_0}{E} \frac{2}{x} \right) \quad (4)$$

where λ_q is the Yukawa coupling for the quark. A similar expression holds for emission from a gluon. There are significant uncertainties in the parton distribution functions as x approaches one, especially for the heavier quarks and gluons. For our conservative bound we include the contributions from only the up and down quarks. The contribution of the remaining quarks and the gluons would only increase the rate.

Ultra-high-energy protons from distant sources have been observed with energies near 10^{20} eV. If this energy is above the threshold energy for Higgs emission, then (4) gives an emission rate from the up and down quarks greater than 10^{-10} eV, corresponding to a distance of a few kilometers. That is, such high energy protons would not travel more than a few kilometers before fragmenting and losing their energy. The observation of these high energy protons is inconsistent with this rapid energy loss. We conclude that the energy of these cosmic protons must be below the threshold for Higgs emission, 10^{20} eV $<$ E_0 . This provides our constraint on positive δ : $\delta < 10^{-18}$.

For negative δ the Higgs is superluminal and anomalous vacuum emission processes by otherwise stable particles are forbidden by energy and momentum conservation. However the modifications of the W and Z propagators lead to potentially observable effects on the behavior of light hadrons and leptons. As in the subluminal case, these effects are largest at high energies and the most stringent constraints come from ultra-high-energy cosmic rays.

Weak decays of charged pions and muons are well described by the tree-level exchange of the W boson. The modification of the W propagator in (2) leads to a modification of the effective interaction. Since $k^2 \sim \mathcal{O}(m_\pi^2, m_\mu^2) \ll M_W^2$ we may safely ignore powers of k^2/M_W^2 . Also, the light quark and leptonic weak currents are nearly conserved and contributions from terms in the vector boson propagators involving k^μ or k^ν are proportional to the tiny light quark and lepton Yukawa couplings. With the neglect of such small effects the effective Lagrangian for the charged-current weak interaction is

$$\mathcal{L}_{wk} = \frac{G_F}{\sqrt{2}} J^\mu J^\nu \left\{ g_{\mu\nu} + n_\mu n_\nu \frac{M_W^2}{(k \cdot n)^2 + E_1^2} \right\} \quad (5)$$

where all reference to the Lorentz violating parameter δ has been subsumed into the energy $E_1^2 \equiv -M_W^2/\delta > 0$. The neutral current weak interaction is similarly modified.

The conventional $g^{\mu\nu}$ term in the interaction involves the square of the weak current's Lorentz invariant length and is proportional to the square of the small meson masses. The Lorentz violating correction, however, involves the time component of this current which grows with energy. Consequently, at high energies this second term becomes much larger than the first and the decay rates of light mesons are greatly enhanced. Notably, the decay is not helicity suppressed at high energy and charged pions decay more often to electrons than muons. An elementary calculation yields a charged pion decay rate to lepton $l = e, \mu$ and an associated neutrino

$$\Gamma = \frac{F_\pi^2}{6\pi E} \left(\frac{G_F M_W^2}{\sqrt{2}} \right)^2 \left(\frac{E^2}{E^2 + E_1^2} \right)^2 K_l \quad (6)$$

where the pion decay constant $F_\pi \sim 130$ MeV, the weak coupling $G_F M_W^2 / \sqrt{2} = g_{\text{wk}}^2 / 8 \sim 1/20$, and $K_l = (1 - m_l^2/m_\pi^2)^2(1 + 2m_l^2/m_\pi^2)$ is close to one for the electron and 0.4 for the muon. A similar expression holds for the rate of muon decay to $e^- \bar{\nu}_e \nu_\mu$ (neglecting the electron and neutrino masses and higher order weak effects):

$$\Gamma = \frac{m_\mu^2}{240\pi^3 E} \left(\frac{G_F M_W^2}{\sqrt{2}} \right)^2 \left(\frac{E}{E_1} \right)^4 g(E/E_1) \quad (7)$$

where $g(x) = 10[3 \arctan(x)/x - 3 + \ln(1 + x^2)]/x^4$ approaches one for small argument.

In a Lorentz invariant world high energy protons impinging on the atmosphere produce a hadronic air shower which may be caricatured as follows. At the highest energies, near 10^{20} eV, hundreds of mesons are usually produced in the initial collision[3]; at these extreme energies the resulting mesons have a greatly dilated lifetime, and charged mesons collide with nuclei in the atmosphere long before they decay. These secondary collisions degrade the energy per hadron and develop the cascade. With the energy degraded the meson lifetime is decreased: some charged mesons in the shower decay above the Earth's surface producing long-lived muons that reach the ground. The neutral pions decay rapidly producing high energy photons. The result is a long cascade (whose length increases with the energy of the primary) containing many muons.

An enhanced decay rate for mesons would radically and visibly alter the morphology of ultra-high-energy cosmic ray proton air showers. At sufficiently high energy even a very small δ would lead to significant Lorentz violation. The charged pion lifetime would be short and these particles would decay before they interact, resulting in high energy charged leptons (about 2/3 electrons and 1/3 muons) accompanied by high energy neutrinos. A

small fraction of the mesons produced in the initial collision are kaons, but they would have a similarly short lifetime, decaying to muons, electrons and neutrinos. If the muon lifetime is also short, high energy muons would decay quickly and not reach the ground. Finally the neutral pion branching fraction to neutrinos would be large, resulting in fewer high energy photons in the shower. The result is a shallow shower with a rapid flow of the energy of the initial proton into electrons and neutrinos: there would no longer be a hadronic cascade.

Pions and muons produced in the initial collision of a primary proton with energy near 10^{20} eV have energies of order or greater than 10^{17} eV. For this energy a value of $\delta = -2 \times 10^{-14}$ gives a charged pion decay length of about 5 meters, a muon decay length of about 5 kilometers, and a neutral pion branching fraction to neutrinos of about 85%. A proton-initiated air shower with these characteristics does not resemble actual ultra-high-energy cosmic ray showers, even in caricature. The observation of conventional high energy showers with their long development length and significant muon component rules out such a large value of $|\delta|$.

A more detailed evaluation of the morphology of ultra-high-energy cosmic ray air showers and the spectrum of charged leptons and neutrinos produced will certainly strengthen the constraints presented here. Similar arguments can be used to constrain other forms of Lorentz violation as well, including cases that do not maintain rotational symmetry.

In a Lorentz invariant world physics depends only on Lorentz covariant quantities: invariants depend only on lengths of four-vectors; rates fall at high energies according to time dilation. But the effects of small departures from Lorentz symmetry generally grow with energy, and the extreme particle energies accessible in cosmic rays provide the ideal environment to seek violations of Lorentz invariance. These experiments may yet provide the first evidence for departures from relativistic invariance.

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